ELASTOPLASTIC RESIDUAL STRESSES IN COMPOSITE BEAMS - A SEMI-ANALYTICAL STUDY

Lucas L. Vignoli (1,2), Paulo P. Kenedi (3)

(1) Center for Nonlinear Mechanics, COPPE, Department of Mechanical Engineering, Universidade Federal do Rio de Janeiro, Brazil

(2) Mechanical Engineering, Universidade Federal do Rio de Janeiro - Campus Macaé, Brazil

(3) Programa de Pós-graduação em Engenharia Mecânica e Tecnologia de Materiais, CEFET/RJ, Brazil.

https://doi.org/10.21452/bccm4.2018.01.03

Abstract

Composite materials have been widely used due to their lightweight capability. For different industrial applications, hybrid beams have been designed to combine the potential advantage of each layer; for instace, for aeronautical structures it is usual to combine Glass Fiber Reinforced Plastic (GFRP) and aluminum layers. In this article a three layer beam is considered, a medial layer is made of GFRP and the other layers are made of aluminum. A detailed discusison about a semi-analytic procedure to evaluate the residual stress in such structure is presented, analising the influence of fiber volume fraction on the GFRP layer and the layers thickness. The results indicate the possibility of growing up plastic residual stresses in the aluminum layers.

1. INTRODUCTION

Combinations of unidirectional GFRP (Glass Fiber Reinforced Plastic) laminas and thin aluminum-alloy plies were used as the composite material in this article. The application of large bending moment loads, for instance, can yield the aluminum plies of a composite beam, generating a residual stresses distribution along its cross section. In this article a semi- analytical approach is proposed, based on Mechanics of Solids, to estimate the cross section elastoplastic residual stress distribution in composite beams submitted to large bending moments. The metallic layers were modelled considering isotropic elastic-perfectly plastic behavior, while GFRP plie was modelled as linear-elastic with transversally isotropy on the fibers plane. The Euler-Bernoulli hypotheses were utilized to develop the proposed analytical model as in Crandall [1] and Vinson and Sierakowski [2].

2. ANALYTICAL MODEL

Considering a symmetric composite beam lay-up with three layers, where the central layer was made of GFRP and the extremities ones were made of aluminum, submitted to pure bending moment, as illustrated in Fig. 1.

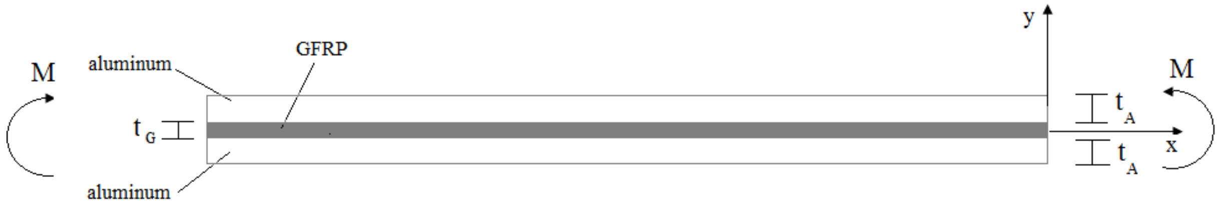


Figure 1: Three layers composite beam

The maximum bending moment applied in this beam is limited, by hyphotesis, to avoid the failure of the GFRP layer. Assuming that GFRP has a smaller strength in compression than in tension Barbero [3], the maximum allowable stress in this layer must be equal to its compressive strength, computed by the following equation as Lo & Chim [4]

$$S_{11}^{c} = \frac{G_G}{1.5 + 12(6/p)^2 (G_G / E_G)} \tag{1}$$

Where E_G and G_G are the longitudinal elastic modulus and the in-plane shear modulus of the GFRP layer, respectively, estimated by

$$E_G = E_f V_f + (1 - V_f) E_m \tag{2}$$

$$G_G = G_m \frac{G_f(1+V_f) + G_m(1-V_f)}{G_f(1-V_f) + G_m(1+V_f)} \tag{3}$$

where V_f is the fiber volume fraction, E_f and E_m are the fiber and matrix longitudinal elastic modulus; G_f and G_m are the fiber and matrix shear modulus.

If the bending moment is such as the whole aluminum layers yield, this bending moment is the maximum that can be applied to avoid the failure of the GFRP layer

$$M_{max}^{ult} = b\left[S_{11}^{c}\frac{t_G^2}{6} + S_y t_A (t_A + t_G)\right] \tag{4}$$

where b is the beam width, t_G and t_A are, respectively, the GFRP and aluminum layers thicknesses and S_y is the aluminum yield strength.

The cross section stress distribution in this limit situation is shown in (5.a)

$$\sigma_{load} = \begin{cases} -E_G \dfrac{M_{max}^{ult}}{EI_{load}} y & if -0.5t_G \le y \le 0.5t_G \\ -S_y sign(y) & otherwise \end{cases} \tag{5.a}$$

$$\sigma_{load} = \begin{cases} -E_G \dfrac{M_{max}}{EI_{load}} y & if -0.5t_G \le y \le 0.5t_G \\ -S_y sign(y) & otherwise \end{cases} \tag{5.b}$$

where $EI_{load} = E_G I_G$ is the equivalent bend stiffness during the load and $I_G = bt_G^3 / 12$ is the second moment of inertia of the GFRP layer.

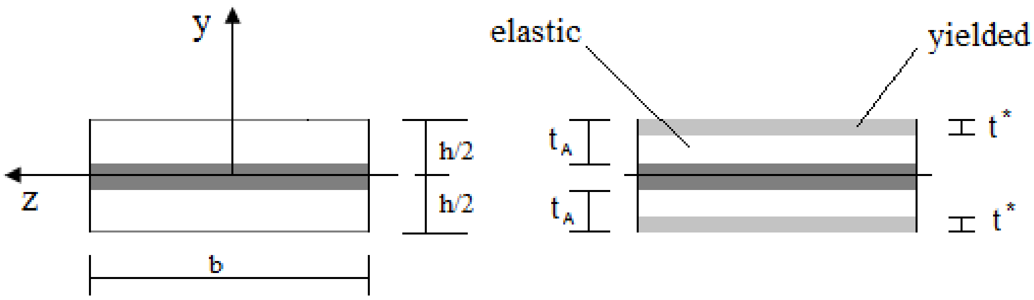
The unload, or springback, is mathematically equivalent to the application of a moment with the same magnitute and oposite direction. Thus, the residual stress is computed using the superposition principle.

In an intermediary situation where the aluminum thicknesses aren't totally yielded $t^* < t_A$ (see Fig. 2), the following equations must be satisfied

$$M_{max} = |k| EI_{unload} + 4S_y bt^* \frac{(h - t^*)}{2} \tag{6}$$

$$|k| = \frac{2S_y}{E_A(0.5h - t^*)} \tag{7}$$

where $EI_{unload} = E_A I_A + E_G I_G$ is the equivalent bend stiffness during the unload and $I_A = b[(h - 2t^*)^3 - t_G^3] / 12$ is the second moment of inertia of the aluminum layer. To obtain the magnitude of $|k|$ and t^*, a simple iterative procedure was implemented in MATLAB. Note that this is the unique numerical step of the present study.



(a) (b)

Figure 2: Composite beam cross section, (a) before loading and (b) after unloading

Once the parameters $|k|$ and t^* are obtained, the stresses of the unload process is estimated:

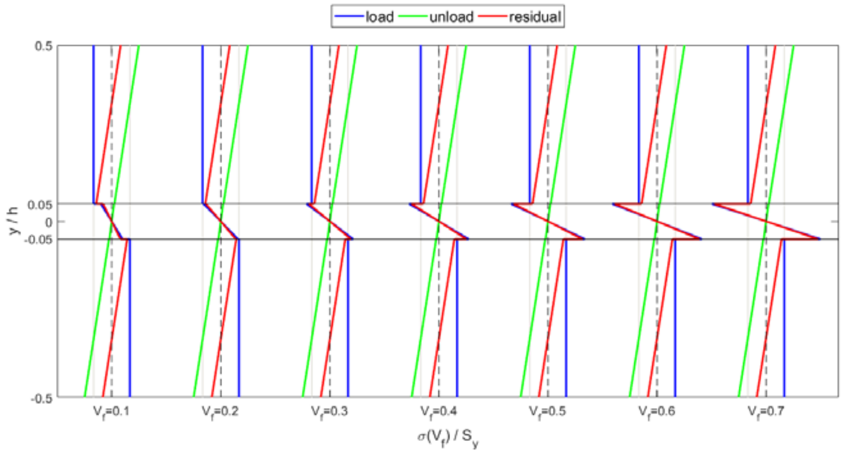
$$\sigma_{unload} = \begin{cases} E_G \dfrac{M_{max}}{EI_{unload}} y & if \quad -0.5t_G \leq y \leq 0.5t_G \\ 2\, S_y sign(y) & if \quad -\left(\dfrac{h}{2} - t^*\right) \leq y \leq \left(\dfrac{h}{2} - t^*\right) \\ E_A \dfrac{M_{max}}{EI_{unload}} y & otherwise \end{cases} \quad (8)$$

Using Eq.(5.b) and Eq.(8), the residual stress cross section distribution can be estimated as

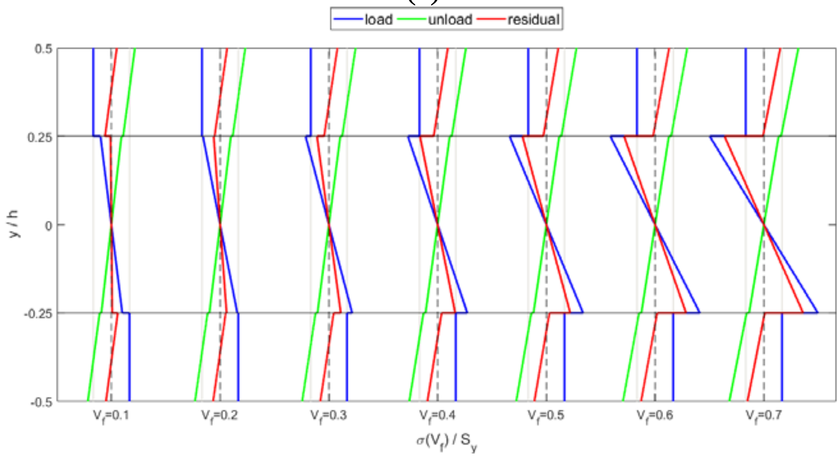
$$s_{residual} = s_{load} + s_{unload} \quad (9)$$

3. RESULTS AND DISCUSSION

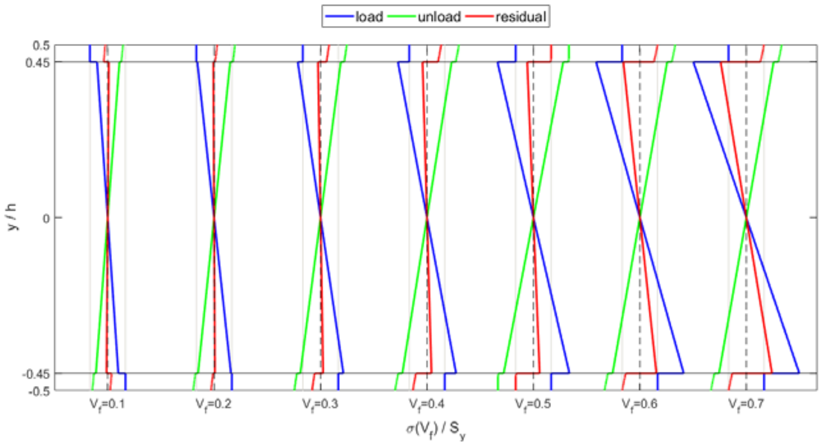
In this analysis, the elastic and shear modulus of the glass fiber and of the epoxy matrix are, respectively: $E_f = 87GPa$ and $G_f = 36.25GPa$; $E_m = 3.2GPa$ and $G_m = 1.18GPa$ as in Kaddour and MJ Hinton [5]. The elastic modulus and the yield strength of the aluminum are $E_A = 72.4GPa$ and $S_y = 345MPa$ as in Abouhamzeh [6]. The beam cross section has the width $b = 10mm$ and height $h = 10mm$. Figure 3 shows the stress distribution along the beam height for different values of fiber volume fractions; each one of them represents a different ratio between the GFRP and aluminum thicknesses.



(a)



(b)



(c)

Figure 3: Stress distribution along the beam height (load, unload and residual) according to: (a) $t_G = 0.1h$, (b) $t_G = 0.5h$ and (c) $t_G = 0.9h$

Note in Fig.3.c the red lines indicate the possibility of residual plastic stress for laminates with thicker GFRP layer and higher fiber volume fraction.

A detailed analysis for the laminate with $t_G = 0.9h$ is presented in Fig.4. In Fig. 4.a shows the contour map of stress for the load application, Fig. 4.b shows the contour map of stress during unload, which is equivalent to a second load, with the same magnitude but with oposite direction and Fig. 4.c shows the contour map of residual stress distribution.

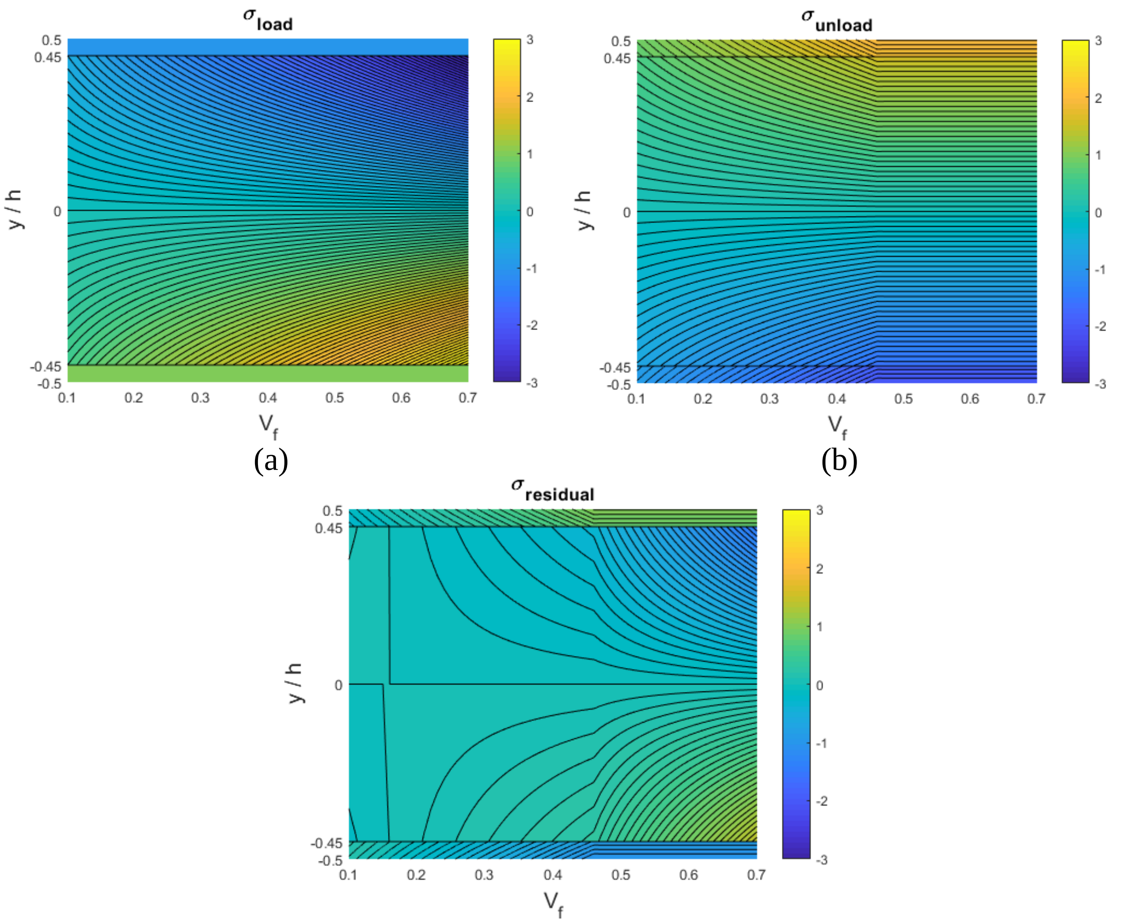


Figure 4: Contour map of the stress distibution ($t_G = 0.9h$): (a) load, (b) unload and (c) residual stresses

Note that the scale of collors was parameterized using σ/S_Y and the whole aluminum layers yield during the load. The horizontal lines on the unload and on the residual maps indicate the yield during the unload step (note for $V_f > 0.45$). As Fig. 4 shows the possibility of plastic residual stress distribution, the main question to be answered is under which conditions the yield during unload could occur.

Figure 5 shows for various combinations of t_G and V_f which ones generate an elastoplastic residual stress distribution along the beam thickness.

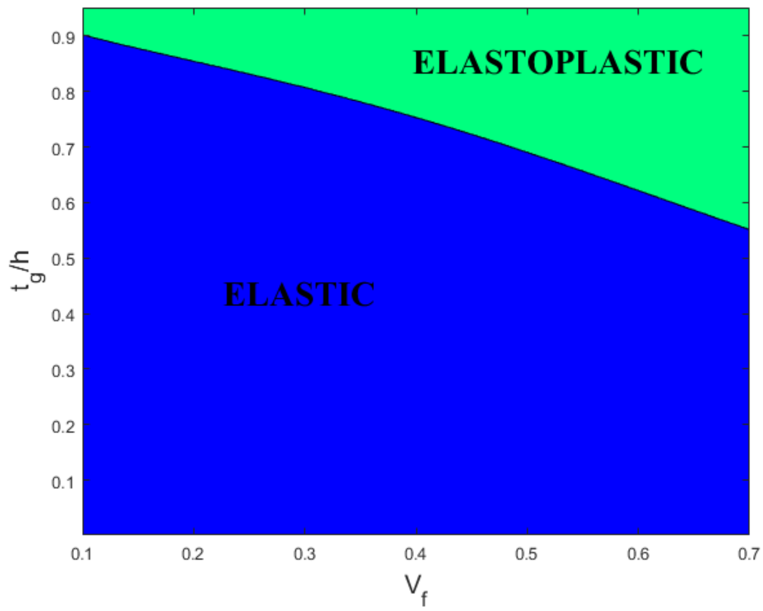


Figure 5: Map indicating for each combinations of V_f and t_G there exist plastic residual stress.

It is possible to recognize that even for very small values of fiber volume fraction it is possible to have a plastic residual stress for high values of GFRP thicknesses.

4. CONCLUSIONS

A semi-analytical model, based in mechanics of solids, was proposed to investigate the behavior of a composite beam, made of plies of aluminum and GFRP, submitted to large value of pure bending moment. Different plies thicknesses and different fiber volume fractions on the GFRP layer were implemented. It was concluded that both variables, fiber volume fraction on the GFRP layer and the layers thickness, can produces significative effects in the beam cross section residual stress distribution. Also, for high bending moments it was verified that the aluminum plies can yield even during unloading.

5. ACKNOWLEDGEMENTS

L.L. Vignoli is grateful for scholarship granted by CAPES.

6. REFERENCES

[1] Crandall, S.H., Dahl, N.C. and Lardner, T.J., ‘An Introduction to the Mechanics of Solids’, Second Edition with SI units (McGraw Hill International Editions, 1978).
[2] Vinson, R. and Sierakowski, R.L., ‘The Behavior of Structures Composed of Composite Materials’, Second Edition (Solid Mechanics and Its Applications, Volume 105, ISBN 978-94-009-5187-7, 1987).
[3] Barbero, E.J., 2018, Introduction to Composite Materials Design, 3 ed., CRC Press.

[4] Lo, K.H., Chim, E.S.M., 1992, "Compressive Strength of Unidirectional Composites", Journal of Reinforced Plastics and Composites, v. 11.

[5] Kaddour, A.S. and Hinton, M.J., 'Input data for test cases used in benchmarking triaxial failure theories of composites', *Journal of Composite Materials* **46** (19–20) (2012), 2295–2312.

[6] Abouhamzeh, M., Sinke, J., Jansen, K.M.B., Benedictus, R. 'Closed form expression for esidual stresses and warpage during cure of composite laminates', *Composite Structures*, **133** (902–910), 2015.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.