INFLUENCE OF SLIPPAGE COEFFICIENT ON THE NON-GEODESIC RETURN TRAJECTORY AT MANDRELS EXTREMITIES IN FILAMENT WINDING PROCESS

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Abstract
Filament Winding (FW) is a manufacturing process for composite materials that winds continuous tows on the surface of a mandrel at a predefined trajectory. This process is usually characterized by forward and backward strokes of a delivery eye that places the tow on the mandrel. Often, the tow's trajectory on the mandrel's surface in each stroke is geodesic, which is the shortest distance between two points on a surface. However, the return procedure at the extremities of the mandrel, since the process is continuous, must follow a non-geodesic path. Differently from the geodesic, which follows the Clairaut relation, non-geodesic trajectories depend on the friction besides on mandrel radius and the defined winding angle. A correct evaluation of such trajectory prevents slippage of the tow or waste of material and time, if the return path is excessively short or long, respectively. Based on differential geometry, the non-geodesic path is herein analytically deduced for a cylindrical surface and numerically determined for revolution surfaces. Examples of application regarding the pattern generation are provided and discussed.

1. INTRODUCTION

Filament winding (FW) is a widespread manufacturing process for composite materials that is usually chosen for the fabrication of revolution parts such as tubes, shafts and pressure vessels. Although simple, the optimal properties are not always obtained due to lack of detailed knowledge of the process. In FW of thermoset composites, a tow of filaments, pre-impregnated (dry) or just impregnated (wet) with a thermoset resin before winding is wound over a rotating mandrel. The tow is led by a delivery eye moving parallel, or near parallel, to the direction of the rotation axis of the mandrel, from one end of the mandrel to the other end, and returning again.

The FW process ends by curing the laminate, formed with the tows, in an autoclave or oven after which the mandrel is withdrawn from the wound part. This task is facilitated by the deposition of a release agent on the mandrel prior to fiber deposition, and this agent may influence the slippage coefficient between tow and mandrel.

The delivery eye positions the tow at a *winding angle*: angle between the tangent to the winding path and the rotational axis, determined through the ratio of the angular velocity of the mandrel and the longitudinal velocity of the delivery eye. Three winding modes are normally mentioned in the literature [1]: polar, hoop and helical winding. In polar winding, applied to manufacture vessels, the tow trajectory goes from one dome to the other, touching the polar opening at each side. In hoop winding, the tow is positioned side by side and the winding angle is near 90°, depending only on mandrel radius and tow width. Hoop winding may be done with a single stroke, which is the longitudinal movement of the delivery eye from one end to the other. The other winding modes must have at least two strokes – forward and backward. In case of helical winding, the tow is positioned at a defined winding angle forming a helix over the mandrel. The movements have to be repetitive in order to obtain uniform surface and thickness and, due to the continuity of the process (as the tow is continuous), a return procedure on the ends of the mandrel has to be adopted.

Many layers may be wound as needed to achieve the desired part thickness, and each layer has a slightly different constitution since the radius increases each time. In helical winding, two entangled layers are simultaneously wound, one at the winding angle $+\alpha$ and the other at $-\alpha$, thus the laminate is antisymmetric.

As aforementioned, the movements of the delivery eye in helical winding are defined by the forward and the backward strokes. Between them, a returning trajectory must be determined. Generally, the strokes follow a geodesic trajectory. i.e. the minimum distance between two points on a generic surface. Such paths are governed by Clairaut relation [3] and are independent of the mandrel-tow friction. In the return regions, however, the tow must follow a non-geodesic trajectory, which is governed by more complex equations that take friction into account.

The returning trajectory impacts product quality, productivity and cost. A short trajectory may generate some slippage, changing the tow's position close to the return region. Depending on the degree of slippage, the regular winding region may also be influenced, yielding a region where the layers are not with opposite winding angles and influencing the mechanical response of the component. On the contrary, a long trajectory reduces slippage but also the process' efficiency regarding cost and time.

Thus, this paper focuses on the analysis and understanding of the returning trajectory characteristics for a more adequate FW processing. The trajectory of tows on surfaces of revolution (both cylindrical and generic) using differential geometry concepts is presented. Analytical solutions are provided for a cylindrical surface and numerical procedures are described for generic surfaces of revolution.

2. ANALYTICAL FORMULATION

2.1 Non-geodesic trajectory for a returning tow on cylindrical surfaces

A generic curve on a surface of revolution is presented and its infinitesimal contribution is shown in Figure 1. L_m and L_p refer to the meridional and the parallel lengths, respectively, while α and θ correspond to the winding angle and the mandrel rotation (both considering the FW process), respectively. z is the direction of the axis of rotation.

Through differential geometry [1], one may define

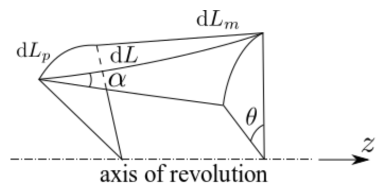


Figure 1- Infinitesimal rotational surface with tow path.

$$\mathrm{d}L_m = \sqrt{E}\mathrm{d}z \qquad \mathrm{d}L_p = \sqrt{G}\mathrm{d}\theta \tag{1}$$

where E and G are the first fundamental forms of the surface. For revolution geometries

$$E = \left(\frac{\mathrm{d}r}{\mathrm{d}z}\right)^2 + 1G = r^2 \tag{2}$$

where r denotes the radius of the surface of revolution. For cylinders, r is constant and thus

$$\mathrm{d}L_m = \mathrm{d}z \qquad \mathrm{d}L_p = r\mathrm{d}\theta \tag{3}$$

and, through trigonometric relations, one obtains

$$\mathrm{d}L = \frac{\mathrm{d}z}{\cos(\alpha(z))} = \frac{r\mathrm{d}\theta}{\sin(\alpha(z))} \Rightarrow r\mathrm{d}\theta = \tan(\alpha(z))\,\mathrm{d}z \tag{4}$$

which implies in a relationship between the variation of the winding angle and the mandrel's rotation. From now on, the dependence of α on z is omitted.

The differential equation of the winding angle variation along the z axis which takes into account the slippage coefficient λ on a generic surface is written as [4,5]

$$\frac{\mathrm{d}\alpha}{\mathrm{d}z} = \lambda\left[\frac{\sin(\alpha)\tan(\alpha)}{r} - \frac{r''\cos(\alpha)}{1+r'^2}\right] - \frac{r'}{r}\tan(\alpha) \tag{5}$$

where r' and r'' correspond to the first and second derivative of the radius in term of the axis of revolution z, respectively. The slippage coefficient in eq. (5) is determined by the equilibrium condition [5] as

$$\lambda = \frac{K_g}{K_N} \tag{6}$$

where K_g and K_N denote the geodesic and normal curvatures, respectively. For cylindrical surfaces, eq. (5) may be simplified as

$$\frac{\mathrm{d}\alpha}{\mathrm{d}z} = \lambda\left[\frac{\sin(\alpha)\tan(\alpha)}{r}\right] \therefore \frac{\mathrm{d}\alpha}{\sin(\alpha)\tan(\alpha)} = \lambda\frac{\mathrm{d}z}{r} \tag{7}$$

Equation (7) describes a non-geodesic path on a cylindrical surface. A geodesic trajectory is obtained considering the slippage coefficient null. In this case, the solution of eq. (5) is the Clairaut relation, defined by

$$r\sin(\alpha) = c \tag{8}$$

where c is a constant. The solution of eq. (7) is

$$\frac{\sin(\alpha_f) - \sin(\alpha_i)}{\sin(\alpha_f)\sin(\alpha_i)} = \frac{\lambda}{r}(z_f - z_i) \Rightarrow \sin(\alpha_f) = \frac{\sin(\alpha_i)}{1 - \frac{\lambda(z_f - z_i)\sin(\alpha_i)}{r}} \tag{9}$$

where α_i and α_f are the initial and final winding angles with their respective position, z_i and z_f, respectively. By eq. (9), one can determine the position where the tow changes the stroke (forward to backward and *vice-versa*), z_f, considering $\alpha_f = 90°$. From a practical point of view, this information is important since it defines the required (minimum) mandrel length.

Another important information that can be extracted from eq. (9) is the rotation angle θ required for the return procedure. By inserting eq. (9) into eq. (4), one obtains

$$r\mathrm{d}\theta = \tan\left(\arcsin\left(\frac{\sin(\alpha_i)}{1 - \frac{\lambda(z_f - z_i)\sin(\alpha_i)}{r}}\right)\right)\mathrm{d}z \tag{10}$$

For integration purposes, let

$$a = \sin(\alpha_i) \qquad b = \lambda/r \qquad \mu = 1 - abz \tag{11}$$

As aforementioned, eq. (9) can be used to define the position on the z-axis where the tow changes its stroke by setting $\alpha_f = 90°$. Considering $z_i = 0$, this leads to

$$z_f = \frac{1-a}{ab} \Rightarrow \mu_f = a \tag{12}$$

Equation (12) measures the required length of the return path at the z-axis. The integrations are carried out by μ, thus the Jacobian is

$$\frac{\mathrm{d}\mu}{\mathrm{d}z} = -ab \tag{13}$$

By inserting eq. (9) into eq. (4) in terms of cosine, one obtains

$$\mathrm{d}L = -\frac{1}{ab\cos\left(\arcsin\left(a/\mu\right)\right)}\mathrm{d}\mu = -\frac{\mu}{ab\sqrt{\mu^2 - a^2}}\mathrm{d}\mu \tag{14}$$

Integration of eq. (14) yields

$$\int_{0}^{L(\mu)} \mathrm{d}L = -\int_{\mu_i}^{\mu} \frac{\mu}{ab\sqrt{\mu^2 - a^2}} \mathrm{d}\mu \;\Rightarrow L(\mu) = \frac{1}{ab}\left[\frac{(a^2 - \mu^2)}{\sqrt{\mu^2 - a^2}} - \frac{(a^2 - 1)}{\sqrt{1 - a^2}}\right] \tag{15}$$

where $\mu_i = 1\ (z = 0)$. By replacing $\mu = a$ into eq. (15), one obtains an indeterminate, since close to the turning point, $\tan(\alpha) \to \infty$.

Applying the L'Hopital rule, one determines the limit of the function with $\mu \to a$, obtaining

$$L(a) = \frac{(1 - a^2)}{ab\sqrt{1 - a^2}} \tag{16}$$

Equation (16) defines the half-length of the returning path (until α=90°). Two limits may be defined: if the initial angle is close to 90°, as in hoop winding, the length is the smallest possible since a ≈ 1, and if α $\approx$ 0°, the half-length required to the return trajectory tends to infinity.

Another important parameter is the required rotation angle θ for the half-length. It may be obtained analogously to L. Firstly, one considers eq. (4) along with eq. (9), thus

$$r\int \mathrm{d}\theta = -\frac{1}{ab}\int \tan\left(\arcsin\left(\frac{a}{\mu}\right)\right)\mathrm{d}\mu \tag{17}$$

which results in

$$\theta(\mu) = \frac{1}{\lambda}\ln\left|\frac{\sqrt{\mu^2 - a^2} - \mu}{\sqrt{1 - a^2} - 1}\right| \tag{18}$$

An interesting point of eq. (18) is that the half-dwell, which is independent on the mandrel radius, is defined as

$$\theta(\mu_f = a) = \frac{1}{\lambda}\ln\left(\frac{a}{1 - \sqrt{1 - a^2}}\right) \tag{19}$$

and, it strongly depends on the winding angle (in a nonlinear fashion), being inversely proportional to the slippage coefficient.

2.2 Non-geodesic trajectory for returning tow on generic surfaces

In order to determine non-geodesic trajectories on generic surfaces, a position vector, $\boldsymbol{d}$, is defined as

$$\boldsymbol{d}(z) = \{r\cos(\theta) \quad r\sin(\theta) \quad z\}^{\mathrm{T}} \tag{20}$$

where both r and θ depend on z. Differentiating $\boldsymbol{r}$ with respect to z, one obtains

$$\frac{\mathrm{d}\boldsymbol{d}}{\mathrm{d}z} = \boldsymbol{d}' = \{r'\cos(\theta) - r\sin(\theta)\,\theta' \quad r'\sin(\theta) - r\cos(\theta)\,\theta' \quad 1\}^{\mathrm{T}} \tag{21}$$

The angle between the vectors $\boldsymbol{r'}$ and $\boldsymbol{\kappa}$ (unit vector in z direction) is the winding angle α.Thus

$$\boldsymbol{d}' \cdot \boldsymbol{\kappa} = |\boldsymbol{d}'||\boldsymbol{\kappa}| \cos(\alpha) \;\Rightarrow\; \cos(\alpha) = \frac{\boldsymbol{d}' \cdot \boldsymbol{\kappa}}{|\boldsymbol{d}'||\boldsymbol{\kappa}|} = \frac{1}{|\boldsymbol{d}'|} \tag{22}$$

Considering that the norm of $\boldsymbol{d'}$ is determined as

$$|\boldsymbol{d}'| = \sqrt{r'^2 + r^2\theta' + 1} \tag{23}$$

One defines the mandrel's rotation by the following differential equation

$$\theta' = \frac{\sqrt{\tan^2 \alpha - r'^2}}{r} \tag{24}$$

which is obtained by algebraic manipulation of eqs. (22) and (23). Equation (24), along with eq. (5), defines the mandrel's rotation and the winding angle through a generic revolution surface. One defines

$$\boldsymbol{H}(z) = \begin{Bmatrix} \theta(z) \\ \alpha(z) \end{Bmatrix} \Leftarrow \boldsymbol{H}'(z) = \begin{Bmatrix} \theta'(z) \\ \alpha'(z) \end{Bmatrix} \tag{25}$$

Due to the characteristics of $\boldsymbol{H}'(z)$, θ and α can be only obtained by a numerical procedure, noting that the last (α) may be solved before the first (θ). Thus, the non-geodesic path over a generic surface of revolution may be determined.

3. RESULTS AND DISCUSSION

Trajectories, winding angle and mandrel's rotation at the return region on cylindrical and surfaces of revolution are presented and discussed in this section. Firstly, concerning cylindrical surfaces, Equation (18) is plotted for four different slippage coefficients ($\lambda = 0.1$, $\lambda = 0.2$, $\lambda = 0.3$ and $\lambda = 0.4$), considering a mandrel with $r = 25$ mm and winding angle of 60° in Figure 2. The curves should be analyzed as at bound to the trajectories. For example, if the slippage coefficient of the mandrel is 0.1, in order to avoid any unwanted displacement of the tow in the return path, the shortest trajectory is presented by the curve with solid circles. Consequently, greater mandrel's rotation and longer trajectories are needed, i.e. a longer mandrel is required.

Figure 2 also implies that the greater the slippage coefficient, the lower the mandrel's rotation and the required size of the return region. Moreover, eq. (18) estimates the minimum required size for the mandrel. For $\lambda = 0.1$, ca. 40-mm long return size is required, while for $\lambda = 0.2$, this length drops to ≈ 20 mm.

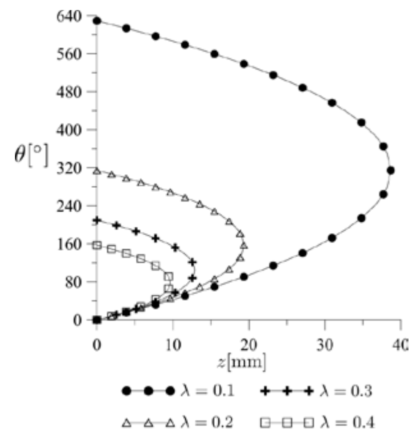
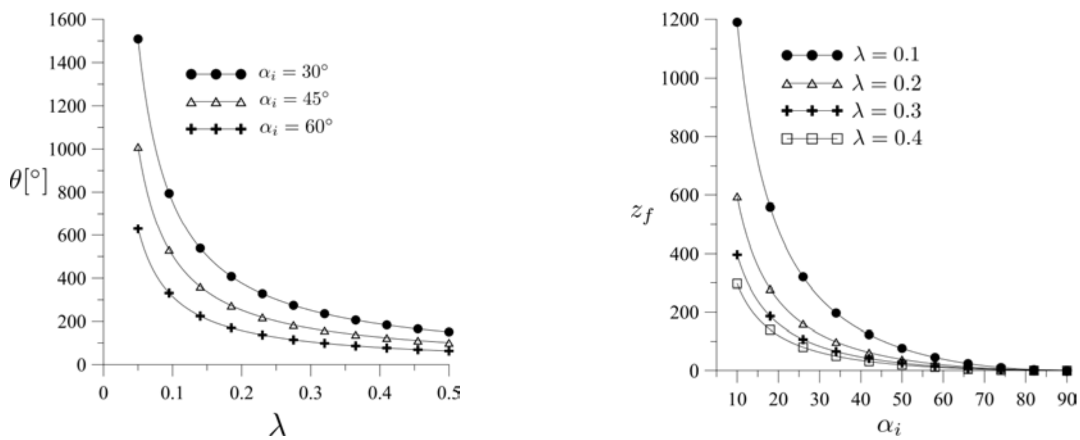


Figure 2 – Slippage coefficient at the returning path: mandrel's rotation and max. z.



(a) (b)

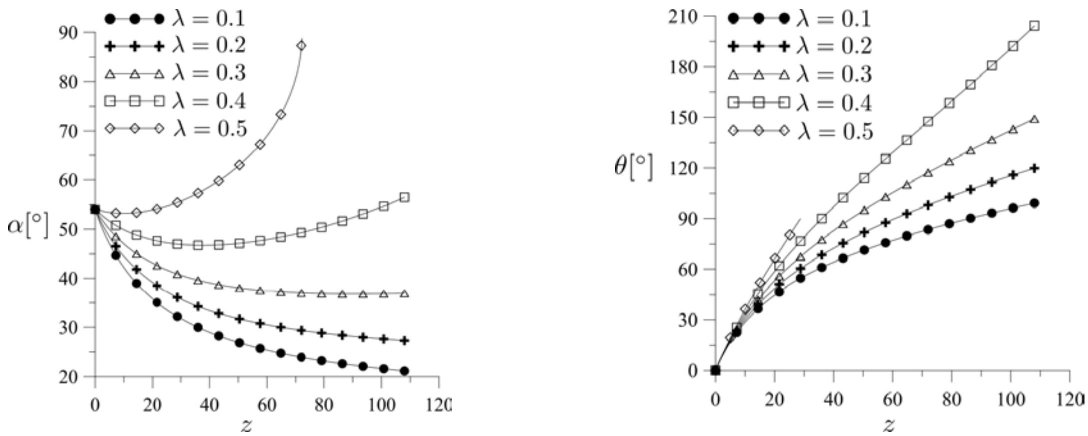
Figure 3 – Bounds for the return region: (a) return angle × slippage coefficient for different winding angles and (b) length of return stroke × winding angle for different slippage coefficients.

The two plots in Figure 2(a)-(b) show the correlation between mandrel's rotation, slippage coefficient and stroke at the return region. Figure 2 also presents the bounds for these parameters and the mandrel's rotation and the final stroke are inversely proportional to the slippage coefficient and the initial winding angle, respectively. As $\lambda \to 0$, the required mandrel's rotation tends to infinity (Figure 3(a)). A similar trend is noted as the winding angle tends to 0°, as mentioned before in the discussion of eq. (16). Also, the closer the winding angle is to 90°, the shorter is the return path.

Figure 4 depicts the winding angle and the mandrel's rotation in a non-cylindrical revolution surface governed by

$$r = 5.5731.10^{-6} z^3 - 0.00226208 z^2 + 0.480329 z + 17.9 \text{ [mm]} \qquad (26)$$

where $z_i = 0$ [mm] and $z_f = 108$ [mm]. Moreover, the winding angle at z_i is 55°. Equation (25) is solved with a Runge-Kutta algorithm – RK4 – with 1500 points. Five slippage coefficients are evaluated. Interestingly, the winding angle for $\lambda = 0.4$ decreases until $\approx 40°$ and then increases to the initial winding angle. The same trend is found for $\lambda = 0.5$, but due to the high friction between the tow and the mandrel, the trajectory has a return ($\alpha = 90°$) before the end of the mandrel, at $z \approx 72$ mm. This behavior is similar to the non-geodesic trajectories in cylindrical surfaces previously shown in Figure 2.



(a) (b)

Figure 4 – Winding angle and mandrel's angle × stroke at the return region for a non-cylindrical revolution surface.

4. CONCLUSIONS

The return region in helical FW is mostly disregarded in published literature work on FW. The return procedure influences process economics, processing time and waste generation, and the pattern formation during regular winding. Through equations $\alpha(z)$ and $\theta(z)$ presented in this work, the non-geodesic path for the return procedure is fully described on a cylindrical surface and also on a generic surface of rotation. The effect of mandrel radius, slippage coefficient and initial winding angle on maximum z and total rotation angle θ was presented, being useful for parameters selection in the FW process.

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