DESIGN OPTIMIZATION OF A COMPOSITE PRESSURE VESSEL USING FEA AND DERIVATIVE BASED METHOD

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https://doi.org/10.21452/bccm4.2018.16.07

Abstract

This paper presents a design optimization, in a thin wall elliptical-headed pressure vessel, considering 2:1 axis rate. The aim is the thickness optimization of the layers of fiber composite material, made of carbon fiber and epoxy resin, using the derivative-based method. It is considered the layup orientation as design variables. Also, the Tsai-Wu failure criterion, evaluated from the stresses calculation by the classical laminate theory, which uses the membrane forces obtained by FEA (Finite Element Analysis), is taken as a constraint for the optimization. In order to reduce computational cost, there is no interaction of the optimization algorithm with the FEA. The stiffness is changed during optimization using constant membrane forces. At the end of the optimization, the membrane forces are recalculated by FEA considering the current rigidity of the structure. The procedure is repeated iteratively until the stress matches the stop criteria. Finally, the safety factor established by ASME is verified, considering a certain class of pressure vessel.

Key-words: Design optimization, Composite materials, Finite element method, Tsai-Wu failure criterion.

1. INTRODUCTION

High-pressure vessels are widely used in commercial and aerospace industries, such as fuel tanks, portable oxygen storage, and compressed natural gas (CNG) pressure vessels for transportation vehicles [1]. On the other hand, the use of fiber reinforced, and polymer-based composites have been increasing. Various numbers of applications have also been flourishing with this development. Parnas and Katirci [2] developed an analytical procedure to design and predict the behavior of fiber-reinforced composite pressure vessels under combined mechanical and hygrothermal loading. Liang *et al.* [3] investigated the optimum design of dome contours for filament-wound composite pressure vessels, subjected to the Tsai–Wu failure criterion and involving problems of the maximum shape factor. The stress field is modeled using classical lamination theory. Pelletier and Vel [4] described a methodology for the multi-objective optimization of fiber reinforced composite materials for strength, stiffness and minimal mass via the layerwise tailoring of fiber orientations and fiber volume fractions. Vafaeesefat [1] presented a new strategy to optimize composite pressure vessels with metallic liners using an adaptive response surface method. The finite analysis is used to evaluate the fitness function and constraints. Romeo *et al.* [5] used a genetic algorithm to optimize the laminate layup to reduce the weight of the tank and ensure that it can resist without at failing catastrophically.

In this paper, a new methodology is proposed for design optimization of complex structures such as a composite pressure vessel. The Finite Element Analysis (FEA) and derivative-based method are used to analyze the pressure vessel. The Tsai–Wu yield criteria are used as a failure criterion for the composite section. The stiffness is modified using constant membrane forces after it is updated and refined optimization. The procedure is repeated iteratively until the stress matches the design criteria. Finally, the safety factor established by ASME is verified, considering a certain class of pressure vessel.

2. MATERIAL AND METHOD

2.1 Material

The composite laminated is symmetrical and quasi-isotropic, with 20 layers and stacking orientation of $[0/45/-45/90/0/45/-45/90/45/-45]_S$. Also, the thickness of 0.6 mm per layer (*i.e.* total thickness of 12 mm). The used material is carbon fiber and epoxy resin H3501 [6], with the properties, shown in Table 1.

Table 1: Material properties of carbon-epoxy H3501 [6].

Elastic properties		**Strength values**	
E_{11} = 138,0 GPa	ν12 = 0,3	X_t = 1447,0 MPa	Y_c = -206,0 MPa
E_{22} = 8,96 GPa	G_{12} = 7,1 GPa	X_c = -1447,0 MPa	S_{12} = 93,0 MPa
ν12 = 0,3	G_{13} = 7,1 GPa	Y_t = 51,7 MPa	

2.2 Optimization Method

The optimization project is divided into external iterations, pre-optimization, and internal optimization. The external iterations aim to update the membrane forces through FEA (finite element analyze) considering the current condition. The pre-optimization, improve the current critical region and aims to reveal another region that becomes critical. Thus, it was considered both regions simultaneously during optimization. Finally, the internal iterations are intended to optimize a laminated composite that meets multiple types of loads. Figure 1(a) illustrates the pressure vessel section analysis, where the Tsai-Wu critical criteria regions are located. The project starts from the minimum thickness required to withstand the internal pressure using an arbitrary prescribed fiber's orientation with a fixed number of layers. The considered pressure vessel in this study has a 1-meter diameter and, it is submitted to an internal pressure of 4.0 MPa. The mesh used for FEA is shown in Figure 1(b). The geometry was modeled with shell elements, the quadrilateral of 8 nodes, with 6 degrees of freedom per node (rotation in x, y and z-axis, and translation in x, y, and z-axis) in the APDL software, the element is called Shell281 (Ansys nomenclature). At the end of the project, in addition to checking the Tsai-Wu failure criterion, the safety factor is evaluated by the quadratic stress interaction criterion established by [7]. The minimization is done using the method of the conjugate gradients. For this, the information of the first derivative is necessary. The objective function is modeled by the internal penalty method,

$$\begin{aligned} &\textbf{\textit{minimize}}\ f(\boldsymbol{x}) \\ &\textbf{such that}\ \ g(\mathbf{x}) \geq 0, j = 1, \dots, n_g, \end{aligned} \tag{1}$$

where $g_j(x)$ is the j-nth constraints and n_g is the total number of constraints. The optimization problem with penalty function [8] is,

$$\textbf{minimize}\ \phi(x,r) = f(\mathbf{x}) + r\sum_{j=1}^{n_g} 1/g_j(\mathbf{x}), \qquad r = r1, r2, \dots, r_i \to 0, r_i > 0 \tag{2}$$

where r is the penalty parameter. The constraints are from the Tsai-Wu failure criterion,

$$f_{TW_j}\left(\boldsymbol{\sigma}_{A_j}\right) < 1, \qquad j = 1, \dots, 20 \tag{3}$$

where $\boldsymbol{\sigma}_{A_j}$ are the stresses in the section A, of the j-nth layer. Thereby,

$$g_j = -f_{TW_j}\left(\boldsymbol{\sigma}_{A_j}\right) + 1 > 0, j = 1, \dots, 20. \tag{4}$$

The optimization has the thickness of the laminate as the cost function. Setting the equations (4) and the cost function in (2), it is possible to obtain the penalty function.

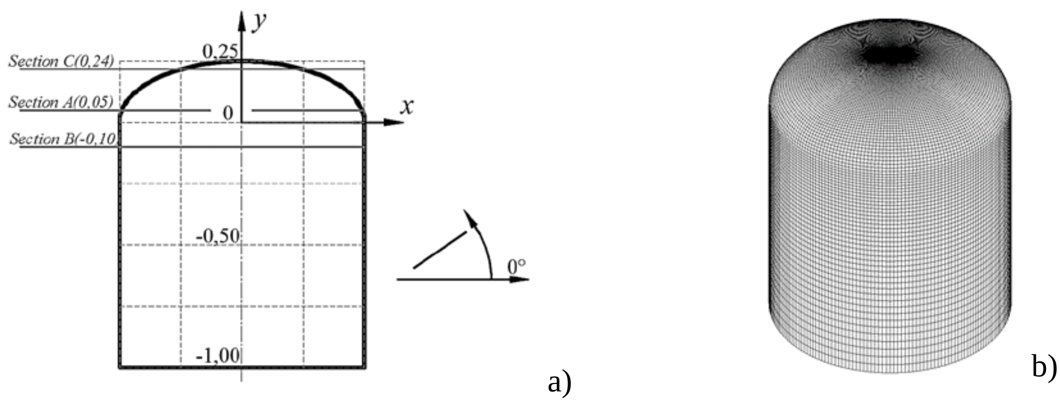


Figure 1: Pressure vessel geometry: a) Arrangement of the vessel in the coordinate system and approximate position of the analyzed sections, b) finite element mesh.

2.3 Failure Criteria

The Tsai-Wu failure criterion represents a factor from the quadratic combination of stresses. In addition, it takes into account the different behavior of resistance in traction and compression [9]. The expression for an orthotropic plane [10] in-plane stress state can be represented by,

$$f_{TW} = F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_{12}^2 + 2F_{12}\sigma_1\sigma_2 < 1 \tag{5}$$

where,

$$\begin{aligned} &F_1 = \frac{1}{X_t} + \frac{1}{X_c}, F_{11} = -\frac{1}{X_c X_t}, F_2 = \frac{1}{Y_t} + \frac{1}{Y_c}, \\ &F_{22} = -\frac{1}{Y_t Y_c}, F_{66} = \left(\frac{1}{S_{12}}\right)^2, F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}} \end{aligned} \tag{6}$$

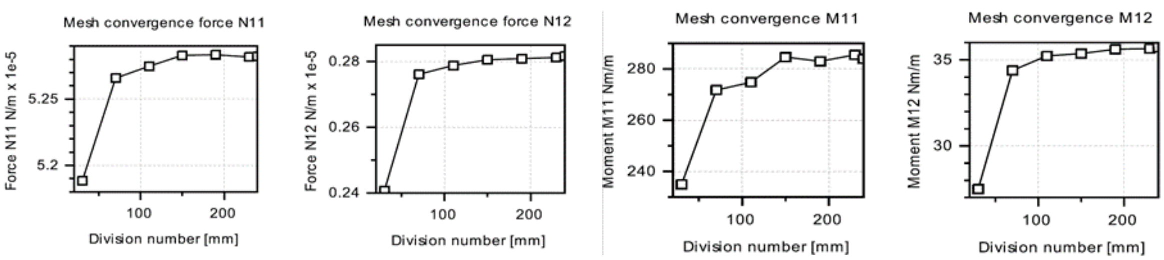
The Quadratic Interaction Strength Criterion designed by [7] is equal to the Tsai-Wu failure criterion and establishes R as a safety factor used in the design methodology by discontinuity analysis. Where must meet R > 1,

$$\mathrm{R}^2(F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_{12}^2) + R(F_1\sigma_1 + F_2\sigma_2) - 1 = 0 \tag{7}$$

where σ_{ij} are the stress in the local coordinate system of the layer.

3. RESULTS AND DISCUSSION

The mesh convergence is based on the interest variable, *i.e.* the membrane forces. Figure 2 shows the convergence of the main membrane forces by the division number in the circumference. The division's number adopted is 200.

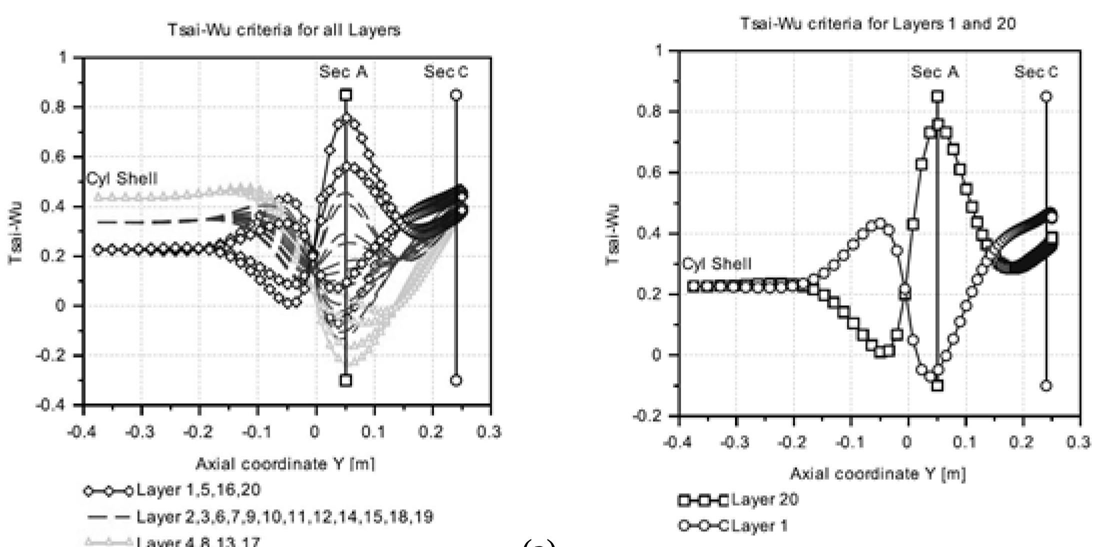


(a) (b) (c) (d)

Figure 2: Mesh convergence for forces: a) N_{11}, b) N_{12}, c) M_{11}, d) M_{12}.

3.1 Preliminary optimization results

When evaluating the Tsai-Wu criterion for the pressure vessel along the axial coordinate, the critical region can be identified. Figure 3(a), illustrates this region marked in the graph as section A, on coordinate y = 0.05 m. The critical value, therefore, occurs on the layer 20. In Figure 3(b), it is illustrated the failure criterion for the layer 20 and its symmetrical layer 1, both at 0°, where loads effect due to the shape transition cylindrical-head can be noted, which justifies the difference of the curves for layers 20 and 1.



(a) (b)

Figure 3: Tsai Wu criteria along the axial coordinate before any optimization: a) Projection of the 20 layers overlapping. b) Projection of the layer 20 and layer 1.

Based on this preliminary analysis, the pre-optimization of the laminate is carried out, taking in consideration the membrane loading that acts in section A. The result of the pre-optimization allows identifying a new critical region, which must be considered simultaneously in the optimization.

3.2 Approximation of the optimal result from constant forces

The failure criterion is evaluated along the pressure vessel for the pre-optimized structure, given that it is possible to identify the new region to be considered. The Figure 4(a), illustrates this region demarcated by section B. With the knowledge of the new critical section, the objective function is modified and the constraints in the eq. (2) become,

$$g_j = \begin{cases} -f_{TW_j}\left(\boldsymbol{\sigma}_{A_j}\right) + 1 > 0 \\ -f_{TW_j}\left(\boldsymbol{\sigma}_{B_j}\right) + 1 > 0 \end{cases} \quad j = 1, \dots, 20. \tag{8}$$

Since the starting point must be in the feasible domain, the optimization is restarted with the modified objective function. The result of the multi-objective optimization is shown in Figure 4(b).

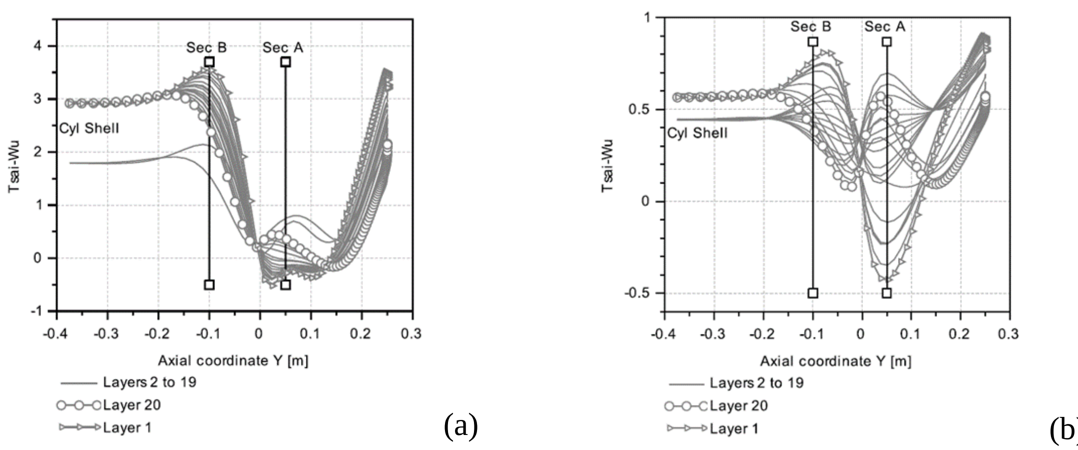


Figure 4: First optimization step, a) single optimized critical region, b) two critical optimized regions simultaneously.

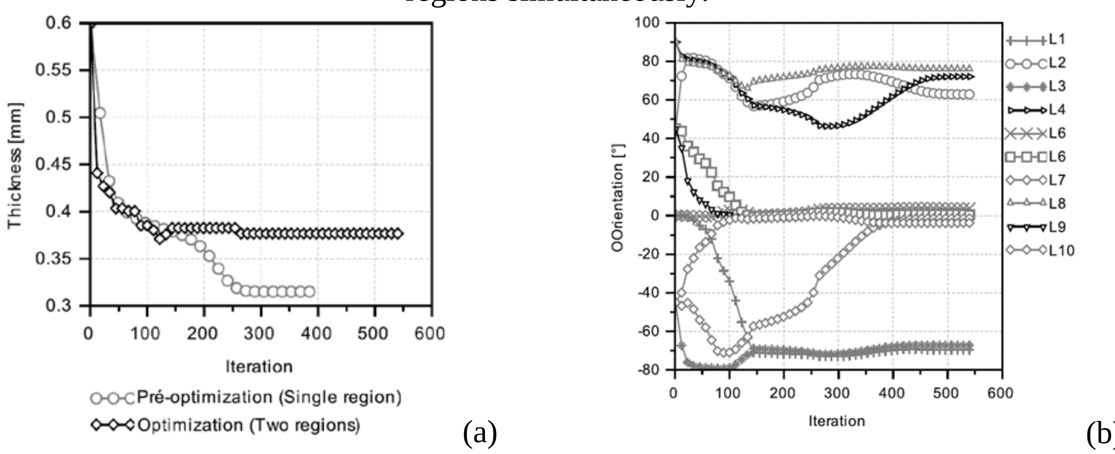


Figure 5: Convergence, a) of thickness for a single region and two regions, b) convergence of the orientations considering both regions.

The stopping criterion is taken from the convergence of the thickness. When only a single critical section is considered, the found thickness is less than when two regions are considered. This is evident since it is easier to satisfy the failure criterion for only one loading condition than for two simultaneously. Figure 5(a) shows the convergence of the thickness for one region and two simultaneously and, Figure 5(b) shows the convergence of slopes as design variables. The result of the first step of the optimization was a total thickness of 7.53 mm or 0.376 mm for each layer with orientation [-70/64/-67/71/5/0/-2/75/-4/4]s. With this result, the geometric properties of the finite element problem were updated, and the membrane forces were obtained for the new structure to refine the optimization.

3.3 Optimization using updated membrane forces

At this stage, the same systematics of the previous step is adopted. Therefore, it is started from the orientations obtained from the current result [-70/64/-67/71/5/0/-2/75/-4/-4], with a thickness of 0.5 mm per layer (10mm in total thickness) and with updated membrane forces. Figure 6(a) shows the Tsai-Wu failure criterion for the optimized structure indicating the two sections A and C, considered simultaneously for the optimization. Figure 6(b) shows the behavior of the thickness' convergence for the pre-optimization and the final optimization. The final total thickness found is 8.15 mm and orientations of [-74/22/-74/69/-2/-2/-2/70/-3/74]s. Finally, the structural problem of finite elements is again updated, where the membrane forces are extracted to the current configuration.

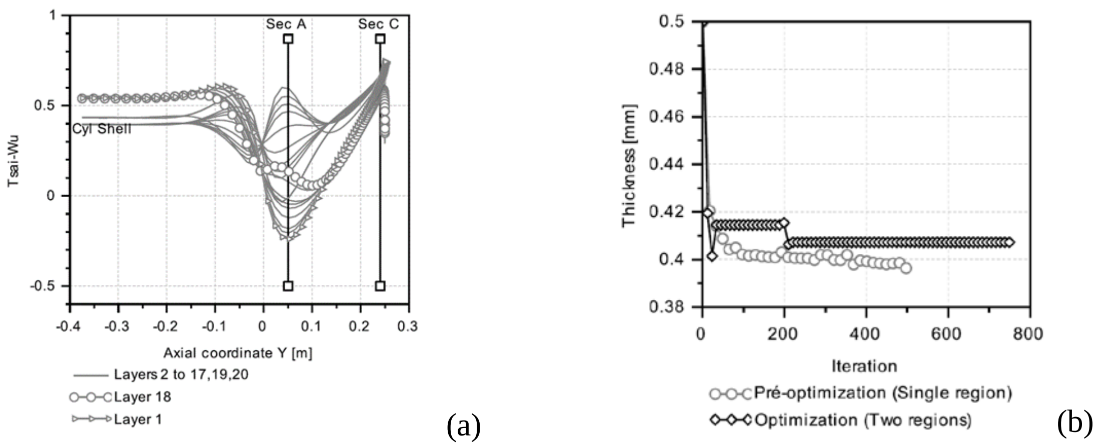


Figure 6: Results of the second stage of optimization. a) Projection of the 20 layers overlapping for Tsai Wu criteria along the y coordinate. b) Convergence of the pre-optimization and optimization.

3.4 Solution analysis

Figure 7 illustrates the convergence for global optimization. Membrane forces N_{11}, N_{22}, N_{12}, and M_{11} have undergone minor changes, on the other hand, the membrane moments M_{22} and M_{12} are in general more sensitive to modification in the structural rigidity. Figure 8 shows the difference between moments M_{22} and M_{12}, for initial and optimized structure. However, the membrane forces used to make the last optimization iteration, are close to the actual forces for the optimized structure except for the moment M_{22} and M_{12}. However, this difference was not enough to make the result unfeasible. This can be verified through the Figure 9, with the new configuration of the laminate and the convergence of the membrane forces verified, the criterion of Tsai-Wu failure is verified for the mesh of the pressure vessel. In addition, it is possible to verify the Quadratic Interaction of Stress Criterion established by [7]. Figure 9 illustrates the failure criteria for the optimal structure, where the two most critical layers 2 and 19 are highlighted. Finally, no section of the vessel exceeds the failure criterion. With this, we have the optimized layup and thickness.

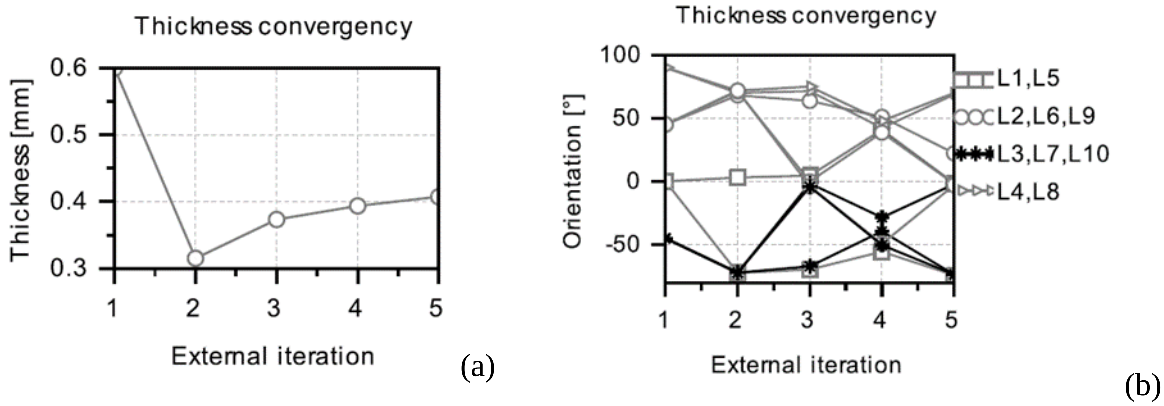


Figure 7: Failure criterion for the optimum design, a) Tsai-Wu, b) Safety factor R by [7].

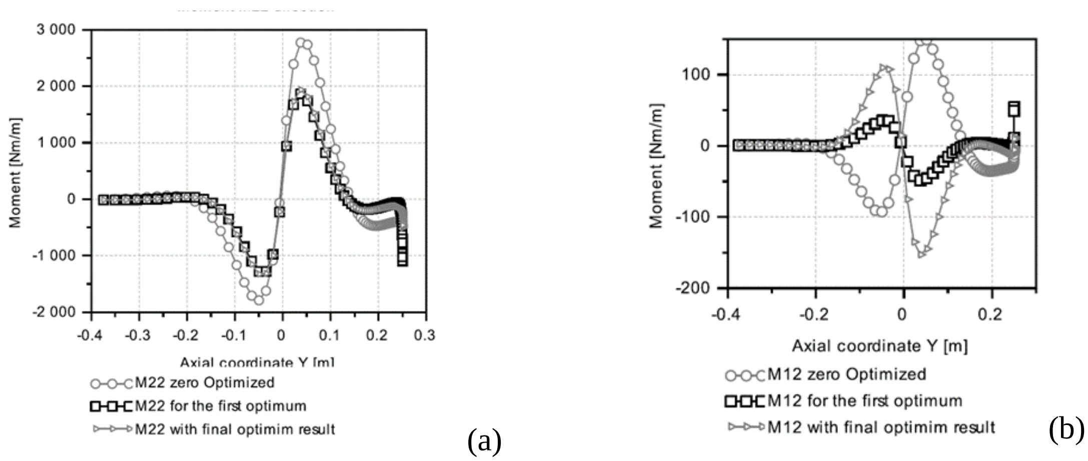


Figure 8: Membrane forces along the optimization: a) Moments M_{22} and b) Moments M_{12}.

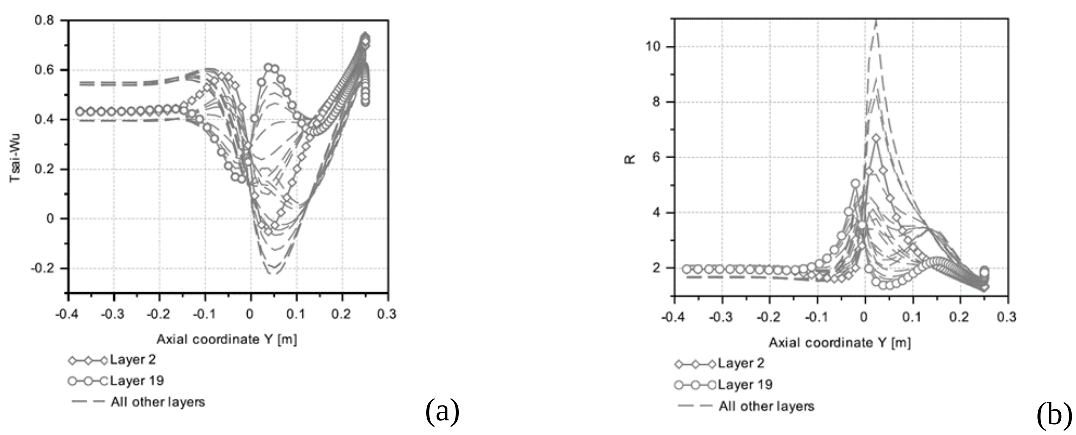


Figure 9: Failure criterion for the optimum design: a) Tsai-Wu, b) Safety factor R by [7].

4. CONCLUSIONS

This paper presented an optimization design in a thin wall pressure vessel under internal pressure. The internal optimization iterations without updating the membrane forces by finite elements bring great advantage due to the substantial reduction of computational cost. Although this procedure, for the time being, is more amenable to restricted problems, where normal membrane stresses N_{xx} is predominant with respect to M_{xx} membrane moments, typical cases of pressure vessels. The agreement with a failure criterion established by a design standard provides credit to evaluate and adjust the study to be used as a methodology for pressure vessel design by analysis. The advantage of using the internal penalty method is that the whole optimization project takes place within the feasible domain, it is, at any time that the optimization is interrupted, there is a feasible optimized result. On the other hand, it impairs the convergence to the global optimum, especially when it is under active constraints. This is a limitation of the internal penalty method. The use of the derivative method yields a future comparison with stochastic optimization methods for this problem nature. The starting point was a lay-up of 0.6 mm per layer or 12.0 mm total thickness, no reduction [0/45/-45/90/0/45/-45/90/45/-45]s, of significant thickness was possible without changing the orientation of the fibers. After optimization, the stacking orientation variables found are [-74/22/-74/69/-2/-2 /-2 /70/-3/74]s and thickness of 0,408 mm per layer or 8,15 mm total, *i.e.* 3.85 mm of total thickness was reduced from the initial laminate. Therefore, this methodology can help in the development of the design of composite pressure vessel.

ACKNOWLEDGMENTS

The authors acknowledge the financial support of the State Research Founding Agency (FAPESC process number: 2017TR1747 and 2017TR784).

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